

Explicit Breaking of $SO(3)$ with Higgs Fields in the Representations $\ell = 2$ and $\ell = 3$

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A gauged $SO(3)$ symmetry is broken into its little groups of the representations $\ell=2$ and $\ell=3$. Explicit Higgs potentials leading to the spontaneous symmetry breaking are constructed. The masses of the gauge bosons and Higgs particles are calculated in terms of the renormalizable potentials. Emergence of Goldstone bosons arising from the absence of certain potential terms is also discussed. Analogous structures between the cosmic strings and disclinations of liquid crystals are noted.

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INTRODUCTION

A general classification of little groups of $SO(3)$ has been given for its irreducible representations[1]. For $\ell = 2$ they are the dihedral groups D_∞ and D_2 , and for $\ell = 3$ representations they are $C_\infty \approx SO(2)$, tetrahedral group T , dihedral group D_3 , the cyclic groups C_3 and C_2 . They have been obtained purely from the algebraic arguments which states that a little group H should have sufficient number of trivial representations in a given irreducible representation of the parent group G . These arguments should be justified by writing explicit potentials and the nature of the parameters associated with the quadratic, third order and quartic terms should be clarified. As we will see not all these closed subgroups of $SO(3)$ are realized as little groups. The potential leading to the little group solutions C_3 and C_2 are identical for the solutions of the tetrahedral group T and the dihedral group D_3 respectively.

In this paper we address ourselves to this problem which is rather important in the liquid crystal phase transitions [2]. There of course, the potential term without gauge interaction plays the principal role where a symmetric tensor field is invoked as an order parameter. In spite of the close resemblance between two problems our main interest rests on the gauge nature of our problem and the role of the Higgs fields in such a theory. In an earlier paper [3], given the representation of $SO(3)$, we have discussed how one can obtain the explicit matrix generators of the little groups. Identification of the Higgs fields receiving non-zero vacuum expectation values for particular little groups was also discussed. In what follows we write down explicit Higgs potentials for each representation $\ell = 2$ and $\ell = 3$ and discuss the minimization conditions which also lead to the masses and associate the fields with real Higgs scalars. We also illustrate the relation between the real scalar Higgs fields and the symmetric tensor fields which seem to be more useful in liquid crystal phenomena [5].

In chapter 2 we briefly discuss the case of $\ell = 2$, a phenomena which is widely known in physics literature [6]. Chapter 3 involves the details of symmetry breaking of $SO(3)$ with the Higgs fields in $\ell = 3$ representation. Finally we discuss our results in Chapter 4 and remark on the possible use of our method for model builders regarding particle physics and cosmology.

GENERAL FRAMEWORK AND THE HIGGS SCALARS IN $\ell=2$ REPRESENTATIONS

The standard Lagrangian of a local gauge theory without fermions is given by

$$L = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{2}(D_\mu\phi)^\dagger(D_\mu\phi) - V(\phi) \quad (1)$$

where the field strengths $F_{\mu\nu}$ and the covariant derivative D_μ are given by

$$\begin{aligned} F_{\mu\nu} &= \partial_\mu W_\nu - \partial_\nu W_\mu + gW_\mu \times W_\nu \\ D_\mu &= \partial_\mu - igW_\mu, \quad W_\mu = \vec{J} \cdot \vec{W}_\mu \end{aligned} \quad (2)$$

where $SO(3)$ generators \vec{J} are the $(2\ell+1) \times (2\ell+1)$ matrices for the irreducible representation ℓ . The Higgs scalars $\phi(\ell m)$ transform like spherical harmonics $Y_{\ell m}$ under the group transformations. We will rather prefer the real scalars

χ_i ($i = 1, 2, \dots, 2\ell + 1$) which can be defined from the complex fields $\phi(\ell m)$, that we will illustrate explicitly when they are needed. A general Higgs potential restricted by renormalizability can be written as[7]

$$V(\chi) = a\chi_i\chi_i + bf_{ijk}\chi_i\chi_j\chi_k + cg_{ijkl}\chi_i\chi_j\chi_k\chi_l. \quad (3)$$

Explicit forms of numerical tensors f_{ijk} and g_{ijkl} depend on the representations. For $\ell = 2$ representation (3) takes the form

$$\begin{aligned} V(\chi) = & a(\chi_1^2 + \dots + \chi_5^2) \\ & + b[\sqrt{3}\chi_1(\chi_3^2 - \chi_4^2) + 2\sqrt{3}\chi_2\chi_3\chi_4 \\ & - (2\chi_1^2 + 2\chi_2^2 - \chi_3^2 - \chi_4^2 - \frac{2}{3}\chi_5^2)\chi_5] \\ & + c(\chi_1^2 + \dots + \chi_5^2)^2 \end{aligned} \quad (4)$$

where the real fields χ_i ($i = 1, \dots, 5$) are related to the Complex Higgs scalars by the relations

$$\begin{aligned} \chi_1 &= \frac{1}{\sqrt{2}}(\phi(22) + \phi(2-2)), & \chi_3 &= \frac{1}{\sqrt{2}}(\phi(21) - \phi(2-1)) \\ \chi_2 &= \frac{i}{\sqrt{2}}(\phi(22) - \phi(2-2)), & \chi_4 &= \frac{i}{\sqrt{2}}(\phi(21) + \phi(2-1)) \\ \chi_5 &= \phi(20). \end{aligned} \quad (5)$$

The potential (4) in terms of the symmetric tensor field T_{ab} ($a, b = 1, 2, 3$), $T_{aa} = 0$ (sum over a is understood) is used to describe the fields in $\ell = 2$ representation. For a further consideration we give the relations between the χ -fields and the components of T_{ab} :

$$\begin{aligned} T_{11} &= \frac{1}{\sqrt{2}}\chi_1 - \frac{1}{\sqrt{6}}\chi_5, & T_{12} &= \frac{1}{\sqrt{2}}\chi_2, \\ T_{22} &= -\frac{1}{\sqrt{2}}\chi_1 - \frac{1}{\sqrt{6}}\chi_5, & T_{23} &= \frac{1}{\sqrt{2}}\chi_4, \\ T_{33} &= \frac{2}{\sqrt{6}}\chi_5, & T_{13} &= \frac{1}{\sqrt{2}}\chi_3, \end{aligned} \quad (6)$$

The potential (4) in terms of the field T_{ab} would read

$$V(T) = aTrT^2 + \sqrt{\frac{8}{3}}bTrT^3 + c[TrT^2]^2. \quad (7)$$

The advantage of (4) is that the potential is expressed in terms of independent fields χ_i whereas (7) includes also dependent fields. Moreover, (4) is more convenient for a gauge theory.

Let us consider the little groups D_2 and D_∞ of $\ell = 2$ in turn. Before we proceed further we discuss the general character of symmetry breaking mechanism for a general potential. The spontaneous symmetry-breaking takes place if the minimality conditions

$$\left\langle \frac{\partial V}{\partial \chi_i} \right\rangle = 0 \quad (8a)$$

$$\left\langle \frac{\partial^2 V}{\partial \chi_i \partial \chi_j} \right\rangle \geq 0 \quad (i, j = 1, 2, \dots, 2\ell + 1) \quad (8b)$$

are satisfied for a set of physical parameters a, b, and c. (8.a-b) determine not only the range of parameter a, b, and c should hold but the eigenvalues of the mass-squared matrix

$$M_{ij}^2 = \frac{1}{2} \left\langle \frac{\partial^2 V}{\partial \chi_i \partial \chi_j} \right\rangle \quad (9)$$

which yield to the masses of the Higgs fields.

The Little group D_2

The $\ell = 2$ representation has two trivial representations of D_2 which can be associated with the fields χ_1 and χ_5 [3]. When these fields take non-zero vacuum expectation values

$$\langle \chi_1 \rangle = v_1 \quad \text{and} \quad \langle \chi_5 \rangle = v_5 \quad (10)$$

it is expected that $SO(3)$ breaks into D_2 . (8a) leads to two sets of independent equations

$$\begin{aligned} av_1 - 2bv_1v_5 + 2cv_1(v_1^2 + v_5^2) &= 0 \\ av_5 + b(v_5^2 - v_1^2) + 2cv_5(v_1^2 + v_5^2) &= 0 \end{aligned}$$

Assuming that both $v_1 \neq 0$ and $v_5 \neq 0$ we obtain

$$bv_1(3v_5^2 - v_1^2) = 0. \quad (11)$$

If $v_1 \neq 0$ and/or $3v_5^2 \neq v_1^2 \neq 0$ are invoked then $b = 0$ should necessarily hold. This choice of vacuum expectation values transform T_{ab} to a diagonal matrix which is equivalent to a $SO(3)$ gauge fixing. As for the eigenvalues of real traceless symmetric matrix they come in two classes, degenerate and non-degenerate cases, which put certain restrictions on the vacuum expectation values v_1 and v_5 . The non-degenerate eigenvalues of T_{ab} can be written as $(1, -1, 0)v_1/\sqrt{2}$ which is equivalent to taking $v_1 \neq 0$ and $v_5 = 0$. Of course, alternative choices $(0, 1, -1)v_1/\sqrt{2}$ and $(-1, 0, 1)v_1/\sqrt{2}$ which restrict v_5 to take $v_5 = \pm\sqrt{3}v_1$. Either of this choice require $b = 0$. Therefore we can work out without loss of generality with the case of $v_1 \neq 0$ and $v_5 = 0$ where the χ_1 field is trivial representation of D_2 but not of D_∞ . This indicates that in breaking $SO(3)$ into D_2 the third order term in potential $V(\chi)$ should be absent. (10a) leads to the condition $v_1^2 = -\frac{a}{2c} > 0$. Then the masses of Higgs fields are

$$m_1^2 = m_2^2 = m_3^2 = m_4^2 = 0, \quad m_5^2 = -2a > 0 \quad (12)$$

which, with $-\frac{a}{2c} > 0$, implies that in the potential (4) we should have $a < 0, b = 0$ and $c > 0$. This is the unique solution for $SO(3) \rightarrow D_2$ breaking. We note that the gauge bosons gain the masses

$$M_{W^\pm} = g\sqrt{\frac{-a}{2c}}, \quad M_{W^0} = 2M_{W^\pm}. \quad (13)$$

Three of the Higgs fields are absorbed by the gauge bosons. Then it is natural to obtain three zero eigenvalues from the mass matrix (9). However we obtain four zero eigenvalues which indicates that one of the Higgs scalars remain as a Goldstone boson while the others gain a mass of $\sqrt{-2a}$.

The little group D_∞ of $\ell=2$

Here only χ_5 transforms as a trivial representation of D_∞ [3]. When we take the vacuum expectation values $\langle \chi_i \rangle = 0, i \neq 5$ and $\langle \chi_5 \rangle = v_5 \neq 0$ we obtain from (8a)

$$a + bv_5 + 2cv_5^2 = 0. \quad (14)$$

The case $v_1 = 0$ and $v_5 \neq 0$ certainly corresponds to the degenerate eigenvalues of T : $(-1, -1, 2)\frac{v_5}{\sqrt{6}}$. The alternative solutions $3v_5^2 = v_1^2$ consistent with $b \neq 0$ are just reshuffling the orders of the eigenvalues as $(2, -1, -1)\frac{v_5}{\sqrt{6}}$ and $(-1, 2, -1)\frac{v_5}{\sqrt{6}}$. They correspond to a definition of a D_∞ trivial representation as a linear combination of χ_1 and χ_5 fields. Therefore it is quite appropriate to work with $v_5 \neq 0$ and $v_1 = 0$. Now the eigenvalues of (9) are obtained as follows

$$\begin{aligned} m_1^2 &= m_2^2 = 0 \\ m_3^2 &= m_4^2 = -3bv_5 > 0 \\ m_5^2 &= v_5(b + 4cv_5) > 0 \end{aligned} \quad (15)$$

It is clear from (15) that two of the Higgs scalars are absorbed by gauge fields giving them the masses

$$M_{W^\pm} = g\sqrt{3}v_5, \quad M_{W^0} = 0. \quad (16)$$

For $ac > 0$, from (14,15,16) we deduce that

$$c > 0, b < 0, a > 0, b^2 \geq 8ac \quad \text{and} \quad v_5 = -\frac{b}{4c} + \frac{\sqrt{b^2 - 8ac}}{4c} > 0 \quad (17)$$

or for $ac < 0$ we simply have $c > 0, a < 0, b < 0$. We note that there is no Goldstone boson in $SO(3) \rightarrow D_\infty$ breaking provided the third order term $b \neq 0$ is included. Otherwise one obtains two Goldstone bosons in addition to a Higgs field with mass $m_5^2 = -2a$.

As we have noted the potential with $b = 0$ leads to the Goldstone boson solutions in both symmetry breaking mechanisms. This is due to the fact that the potential with $b = 0$ has a global $SO(5)$ symmetry larger than $SO(3)$ gauge symmetry[4].

BREAKING $SO(3)$ BY THE HIGGS SCALARS OF $\ell=3$

A completely symmetric tensor $T_{abc} = 0(a, b, c = 1, 2, 3)$ of rank 3 with the trace condition $T_{aab} = 0$ can be used to describe the Higgs scalars. Although the symmetric tensor has more practical use in the liquid crystal phenomena [5] it is not very convenient for our calculations. We rather prefer working with the $\chi_i (i = 1, 2, \dots, 7)$ fields for $\ell = 3$ representation. The χ_i scalars can be defined from the Higgs fields $\phi(\ell m)$:

$$\begin{aligned} \chi_1 &= \frac{1}{\sqrt{2}}(\phi(33) - \phi(3-3)), & \chi_4 &= \frac{i}{\sqrt{2}}(\phi(32) - \phi(3-2)) \\ \chi_2 &= \frac{i}{\sqrt{2}}(\phi(33) + \phi(3-3)), & \chi_5 &= \frac{1}{\sqrt{2}}(\phi(31) - \phi(3-1)) \\ \chi_3 &= \frac{1}{\sqrt{2}}(\phi(32) + \phi(3-2)), & \chi_6 &= \frac{i}{\sqrt{2}}(\phi(31) + \phi(3-1)) \\ \chi_7 &= \phi(30). \end{aligned} \quad (18)$$

It is not difficult to express the components of the symmetric traceless tensor in terms of the χ -fields

$$\begin{aligned} T_{112} &= -\frac{1}{2}\chi_2 + \frac{1}{2\sqrt{15}}\chi_6, & T_{223} &= \frac{1}{\sqrt{6}}\chi_3 + \frac{1}{\sqrt{10}}\chi_7, \\ T_{113} &= -\frac{1}{\sqrt{6}}\chi_3 + \frac{1}{\sqrt{10}}\chi_7, & T_{233} &= -\frac{2}{\sqrt{15}}\chi_6, \\ T_{122} &= \frac{1}{2}(\chi_1 + \frac{1}{\sqrt{15}}\chi_5), & T_{123} &= -\frac{1}{\sqrt{6}}\chi_4, \\ T_{133} &= -\frac{2}{\sqrt{15}}\chi_5. \end{aligned} \quad (19)$$

The other three components of tensor field T_{111} , T_{222} , and T_{333} can be obtained from the vanishing of trace of the tensor field. It is easier to write down the potential with the tensor field which reads

$$\begin{aligned} V(T) &= aT_{ijk}T_{ijk} + b(T_{ijk}T_{ijk})^2 \\ &\quad - 6c[19T_{ijk}T_{ij\ell}T_{mnk}T_{mn\ell} + 44T_{ijk}T_{i\ell m}T_{j\ell n}T_{kmn}]. \end{aligned} \quad (20)$$

Note that no third order invariant polynomial exists for $\ell=3$ potential but we have two independent fourth order polynomials. In terms of the χ -fields the potential (20) reads explicitly

$$\begin{aligned}
V(\chi) = & a(\chi_1^2 + \cdots + \chi_7^2) + b(\chi_1^2 + \cdots + \chi_7^2)^2 + c[9(\chi_1^2 + \chi_2^2)^2 \\
& - 16(\chi_3^2 + \chi_4^2)^2 + 18(\chi_1^2 + \chi_2^2)(\chi_3^2 + \chi_4^2) \\
& - 42((\chi_1^2 + \chi_2^2)(\chi_5^2 + \chi_6^2) - 2(\chi_3^2 + \chi_4^2)(\chi_5^2 + \chi_6^2) \\
& - 72((\chi_1^2 + \chi_2^2)\chi_7^2 + 48(\chi_3^2 + \chi_4^2)\chi_7^2 - 16\sqrt{15}\chi_4\chi_5\chi_6\chi_7 \\
& + 5(\chi_5^2 + \chi_6^2)^2 + 20\sqrt{15}(\chi_1\chi_3\chi_5 - \chi_2\chi_4\chi_6 - \chi_1\chi_4\chi_5 + \chi_2\chi_3\chi_6) \\
& + 24\sqrt{15}(\chi_1\chi_5\chi_6^2 - \chi_2\chi_5^2\chi_6) + 8\sqrt{15}(\chi_3\chi_6^2\chi_7 - \chi_3\chi_5^2\chi_7) \\
& + 8\sqrt{15}(\chi_1\chi_5^3 + \chi_2\chi_6^3) + 40\sqrt{15}(\chi_1\chi_3\chi_4\chi_6 + \chi_2\chi_3\chi_4\chi_5) \\
& + 120(\chi_1\chi_3\chi_5\chi_7 + \chi_2\chi_3\chi_6\chi_7 + \chi_2\chi_4\chi_5\chi_7 - \chi_1\chi_4\chi_6\chi_7)].
\end{aligned} \tag{21}$$

We now, in turn, discuss the possible little group candidates $SO(2) \approx C_\infty, D_3, T, C_3$ and C_2 of $\ell = 3$ representation of $SO(3)$.

The little group $SO(2)$ of $\ell=3$

In ref.[3] We had chosen a representation where χ_7 transforms as a trivial representation of $SO(2)$. To break $SO(3)$ into $SO(2)$ all the fields $\langle\chi_a\rangle = 0$ ($a \neq 7$) take zero expectation values except $\langle\chi_7\rangle = v_7 \neq 0$. The minimality of the potential (8) leads to the results

$$v_7^2 = -\frac{a}{2b} > 0 \tag{22}$$

$$\begin{aligned}
m_1^2 &= m_2^2 = \frac{36ac}{b}, & m_3^2 &= m_4^2 = -\frac{24ac}{b} > 0 \\
m_5^2 &= m_6^2 = 0, & m_7^2 &= -2a > 0
\end{aligned} \tag{23}$$

where m_a^2 ($a = 1, \dots, 7$) are the eigenvalues of the mass matrix (9). It is obvious from (22-23) that no parameters exist satisfying the positivity of masses. Therefore $SO(2)$ is not a little group of $\ell = 3$ representation when $a \neq 0$, $b \neq 0$ and $c \neq 0$. For $a < 0$, $b > 0$ and $c = 0$ $SO(3) \rightarrow SO(2)$ breaking is possible. However we obtain four Goldstone bosons in this latter case. We also note here that the potential with $c = 0$ has a global symmetry $SO(7)$ which leads to the Goldstone boson solutions. This is a general case for the potential with $c = 0$ which will repeat in the following sections.

The little group D_3

For this little group [3] it is the χ_1 field which transforms as a trivial representation of D_3 . Assigning the vacuum expectation values $\langle\chi_1\rangle = v_1$ we check the minimality conditions in (8) and obtain masses of Higgs fields using (9)

$$v_1^2 = -\frac{a}{2(b+9c)} > 0 \tag{24}$$

$$\begin{aligned}
m_1^2 &= -2a > 0, & m_2^2 &= m_3^2 = m_4^2 = 0 \\
m_5^2 &= m_6^2 = -\frac{30ac}{b+9c} > 0, & m_7^2 &= -\frac{45ac}{b+9c} > 0.
\end{aligned} \tag{25}$$

It is certain that the Higgs fields χ_2, χ_3 and χ_4 are absorbed by the gauge bosons giving them the masses

$$M_{W^\pm} = g\sqrt{3}v_1, \quad M_{W^0} = \sqrt{6}M_{W^\pm}. \tag{26}$$

From (26) and (25) we obtain the relations

$$a < 0, \quad b + 9c > 0 \quad \text{and} \quad c < 0. \quad (27)$$

The relations in (26) indicate that two Higgs fields χ_1 and χ_7 transforming as singlets under D_3 gain different masses while χ_5 and χ_6 transforming as a doublet of D_3 get the same mass as expected. The minimum of the potential takes the value $V_{\min} = -\frac{a^2}{4(b+9c)}$.

Two different cases need to be discussed in this breaking:

i) $a \neq 0, b \neq 0, c = 0$

This shows that the symmetry breaking is possible but one will be left with one Higgs field and three Goldstone bosons.

ii) $a \neq 0, b = 0, c \neq 0$

The potential does not possess minimum in this case. Consequently no symmetry breaking takes place at all. Therefore parameters a, b , and c should satisfy the conditions (27) which lead to physical theory in $SO(3) \rightarrow D_3$ breaking.

The Tetrahedral group T as a little group of $\ell=3$ representation

The $\ell = 3$ representation of $SO(3)$ is the lowest dimensional representation which admits the tetrahedral group T as a little group of $SO(3)$. In the representation of ref.[3] χ_4 is the only scalar field which transforms as a trivial representation of the tetrahedral group T . When it gains non-zero expectation value $\langle \chi_a \rangle = 0$ ($a \neq 4$) the $SO(3)$ is expected to break into the subgroup T .

The minimality conditions (8) and the eigenvalues of Higgs masses lead to the results

$$v_4^2 = -\frac{a}{2(b-16c)} > 0 \quad (28)$$

$$\begin{aligned} m_1^2 &= m_2^2 = m_3^2 = 0, \\ m_4^2 &= m_5^2 = m_6^2 = -\frac{40ac}{b-16c} > 0, \quad m_7^2 = -2a > 0. \end{aligned} \quad (29)$$

Here what we observe that the squared-mass matrix is not diagonal and those in (30) are just the eigenvalues of the matrix where the associated fields are the linear combinations of $\langle \chi_a \rangle$ fields. They were indeed listed in ref. [3] and we don't see any reason to reproduce them here. The (29) and (30) are satisfied provided we have

$$a < 0, \quad b - 16c > 0 \quad \text{and} \quad c > 0. \quad (30)$$

Three of the Higgs fields are absorbed by the gauge bosons which gain equal masses

$$M_{W^\pm} = M_{W^0} = \sqrt{2}gv_4. \quad (31)$$

The remaining three Higgs fields which transform as a three-dimensional representation of T gain equal masses $\sqrt{\frac{40ac}{-b+16c}}$, while the last field which transforms as a non-trivial singlet gain the mass $\sqrt{-2a}$. It is quite interesting to note that (31) is different from (28) in the sense that $c > 0$ in (31) but $c < 0$ in the latter. That is, in the $SO(3) \rightarrow D_3$ breaking c is strictly negative whereas in $SO(3) \rightarrow T$, c is strictly positive. Positive quartic terms are to be present in the potential in order to obtain a $SO(3)$ breaking into the tetrahedral group T .

If one ignores the last fourth order term ($c = 0$) in the potential which possesses a global $SO(7)$ symmetry, the symmetry breaking is possible but with three Goldstone bosons and a Higgs field. If we simply delete the second term ($b = 0$) then the symmetry breaking is not possible.

The little group C_3 ?

The $\ell = 3$ representation has three trivial singlet representations of C_3 . In a particular choice of representations of C_3 generators the fields χ_1 , χ_2 , and χ_7 transform as a trivial representation of C_3 . When these fields take the non-zero expectation values $\langle \chi_a \rangle = v_a \neq 0$ ($a = 1, 2, 7$) while the others receive zero expectation values then (8a) leads to the relations

$$\begin{aligned} v_1^2 + v_2^2 &= -\frac{2a}{9(b-16c)} > 0 \\ v_7^2 &= -\frac{5a}{18(b-16c)} > 0. \end{aligned} \quad (32)$$

the square-mass matrix (9) in this case is more complicated. Nevertheless the eigenvalues of the matrix turn out to be exactly the same as those of the matrix obtained in the case of tetrahedral group T , namely

$$\begin{aligned} m_1^2 &= m_2^2 = m_3^2 = 0, \quad m_7^2 = -2a > 0, \\ m_4^2 &= m_5^2 = m_6^2 = -\frac{40ac}{b-16c} > 0 \end{aligned}$$

which imply, together with (33), the range of parameters $a < 0$, $b - 16c > 0$ and $c > 0$. The minimum of the potential is also the same as that of tetrahedral group

$$V_{\min} = -\frac{a^2}{4(b-16c)} \quad (33)$$

The C_3 being a subgroup of the tetrahedral group T both groups have the identical solutions. This follows from the fact that $\nu_1^2 + \nu_2^2 + \nu_7^2 = \nu_4^2 = -\frac{a}{2(b-16c)}$. If one applies to the solution for the little group T a rotation leaving the above equation invariant the result is precisely the same solution for C_3 . Therefore C_3 is not a little group of $SO(3)$ gauge symmetry for the potential of $\ell = 3$ representation.

The little group C_2 ?

In this case $\ell = 3$ irreducible representation possesses three trivial representations of C_2 . In a special embedding of C_2 in $SO(3)$ we can associate the fields χ_3 , χ_4 , and χ_7 with the trivial representations. By assigning the vacuum expectation values $\langle \chi_a \rangle = v_a \neq 0$ ($a = 3, 4, 7$) we obtain the solutions

$$\begin{aligned} v_3^2 + v_4^2 &= -\frac{3a}{16(b+9c)} > 0 \\ v_7^2 &= -\frac{5a}{16(b+9c)} > 0 \end{aligned} \quad (34)$$

All gauge bosons gain masses. The square mass matrix (9) has the eigenvalues

$$\begin{aligned} m_1^2 &= m_2^2 = m_3^2 = 0, \quad m_4^2 = m_5^2 = \frac{30ac}{(b+9c)} > 0, \\ m_6^2 &= -\frac{45ac}{(b+9c)} > 0, \quad m_7^2 = -2a > 0 \end{aligned} \quad (35)$$

(36) and (37) are satisfied $a < 0$, $b + 9c > 0$ and $c < 0$ and the minimum of the potential takes place at a value $V_{\min} = -\frac{a^2}{4(b+9c)}$. Therefore $SO(3) \rightarrow C_2$ breaking produces the same results of $SO(3) \rightarrow D_3$ breaking except that the expectation values of the fields are totally different. Absence of the quartic term ($c = 0$) would yield a theory with one Higgs fields and three Goldstone bosons. Also here, we encounter with a problem similar to the case of $C_3 - T$ symmetry. If one applies a rotation to the solution for the little group D_3 one can obtain the solution for the group C_2 where $\nu_1^2 = \nu_3^2 + \nu_4^2 + \nu_7^2 = -\frac{a}{2(b+9c)}$. This proves that the C_2 is not a little group.

CONCLUSIONS

Breaking a gauged $SO(3)$ into its discrete subgroups has been studied with renormalizable potentials of Higgs fields for the irreducible representations $\ell = 2$ and $\ell = 3$. For the representation $\ell = 2$ the little groups are D_∞ and D_2 . The third order potential should be absent for a D_2 breaking while D_∞ requires the presence of all terms in the potential. Masses of Higgs and gauge bosons have been calculated in terms of the parameters of the potential. An interesting phenomenon is that in the breaking $SO(3) \rightarrow D_2$ one finds a Goldstone boson as well as three Higgs particles, two of which have equal masses.

Breaking $SO(3)$ into little groups of $\ell = 3$ representation turns out to be very interesting. For this representation third order potential term is absent, instead one has two independent quartic polynomials in the potential. One of the quartic term can be arranged as the square of the quadratic term. We have calculated the Higgs particle masses as well as the masses of the gauge bosons in terms of the parameters of the Lagrangian. What we have observed is that the absence of the quartic polynomial which is not square of the quadratic term leads to presence of Goldstone bosons. Breaking a gauged $SO(3)$ into discrete subgroups without Goldstone bosons are possible only with the presence of two independent quartic terms.

What we have not dealt with is the nature of non-abelian cosmic strings arising from such breakings. Since $SO(3)$ can be embedded in a GUT candidate such as E_6 and E_8 through one of their $SU(3)$ subgroups the technique we have discussed could be very useful for model builders, in particular, for those who wish to study the role of non-abelian cosmic strings in cosmological models. We had already checked that breaking $SU(3)$ through its $SO(3)$ subgroup with Higgs in its adjoint representation results in abelian cosmic strings. This encourages us to study E_6 breaking from the point of view of cosmic string formation, a phenomena which has not been worked out so far in the literature.

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